



Describing and Evaluating Algorithms

+ Computer programs

- **procedures**
 - called
 - with **parameters** (inputs)
 - **return** a value (output)
- **array**
 - aggregate of data of the same type in one entity
 - each **entry**
 - has an **index** (indices start at 1)
 - has an **element** (value)
 - square brackets [] indicate the index of an array
 - for instance the i -th element of array A is written $A[i]$

+ Example Procedure Linear Search

Procedure LINEAR-SEARCH(A, n, x)

Inputs:

- A : an array
- n : the number of elements in A to search through
- x : the value being searched for

Output: Either an index i for which $A[i] = x$, or the special value NOT-FOUND, which could be any invalid index into the array, such as 0 or any negative integer.

1. Set $answer$ to NOT-FOUND.
2. For each index i , going from 1 to n , in order:
 - A. If $A[i] = x$, then set $answer$ to the value of i .
3. Return the value of $answer$ as the output.

+ Example Procedure Better Linear Search

Procedure BETTER-LINEAR-SEARCH(A, n, x)

Inputs:

- A : an array
- n : the number of elements in A to search through
- x : the value being searched for

Output: Either an index i for which $A[i] = x$, or the special value NOT-FOUND, which could be any invalid index into the array, such as 0 or any negative integer.

1. For each index i , going from 1 to n , in order:
 - A. If $A[i] = x$, then return the value of i as output.
2. Return NOT-FOUND as the output.

+ Example Procedure Better Linear Search

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1. For each index i , going from 1 to n , in order:
 - A. If $A[i] = x$, then return the value of i as output.
2. Return NOT-FOUND as the output.

Can we do better? How?

+ How can we do better?

- **Notice**
 - we do 2 comparisons every time
- **Question**
 - can we do just one comparison each time?

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- Insight
 - we know it stops if it's the item is in the array
- Answer
 - replace the last value with a sentinel

+ Sentinel? Huh?

- A sentinel is the value that will stop a while loop, but is not the 'true' item searched for.
- Let's say you have $X = \text{"ABCDE"}$ and you are searching for 'G'
 - set last to 'E' set $X[n-1]$ to 'G'
 - resulting in $X = \text{"ABCDG"}$ and last = 'E'
- Now you know that you will find 'G'
- but you still have to do at least one additional test.

+ Example Procedure Sentinel Linear Search

Procedure SENTINEL-LINEAR-SEARCH(A, n, x)

Inputs and Output: Same as LINEAR-SEARCH

1. Save $A[n]$ into *last* and then put x into $A[i]$.
2. Set i to 1.
3. While $A[i] = x$, do the following
 - A. Increment i .
4. Restore $A[n]$ from *last*.
5. If $i < n$ or $A[i] = x$, then return the value of i as the output.
6. Otherwise, return NOT-FOUND as the output.

+ Analyzing time complexity

- Theta notation
- Big O notation
- little o notation

+ Showing correctness

- Using Loop invariants
 - Initialization – invariant is true before first iteration
 - Maintenance – as long as it is true before the first iteration, the invariant is also true before the next iteration
 - Termination – the loop ends, and when it does, the loop invariant along with the reason that the loop terminated, gives us a useful property
- Using Recursion
 - Induction
 - prove true for base case(s)
 - assuming true for n (or $n-1$), prove true for $n+1$ (or n)

