Statistical Properties of Text

- How is the frequency of different words distributed?
- How fast does vocabulary size grow with the size of a corpus?
- Such factors affect the performance of information retrieval and can be used to select appropriate term weights and other aspects of an IR system.

Word Frequency

- A few words are very common.
  - 2 most frequent words (e.g. “the”, “of”) can account for about 10% of word occurrences.
- Most words are very rare.
  - Half the words in a corpus appear only once, called *hapax legomena* (Greek for “read only once”)
- Called a “heavy tailed” distribution, since most of the probability mass is in the “tail”

Sample Word Frequency Data

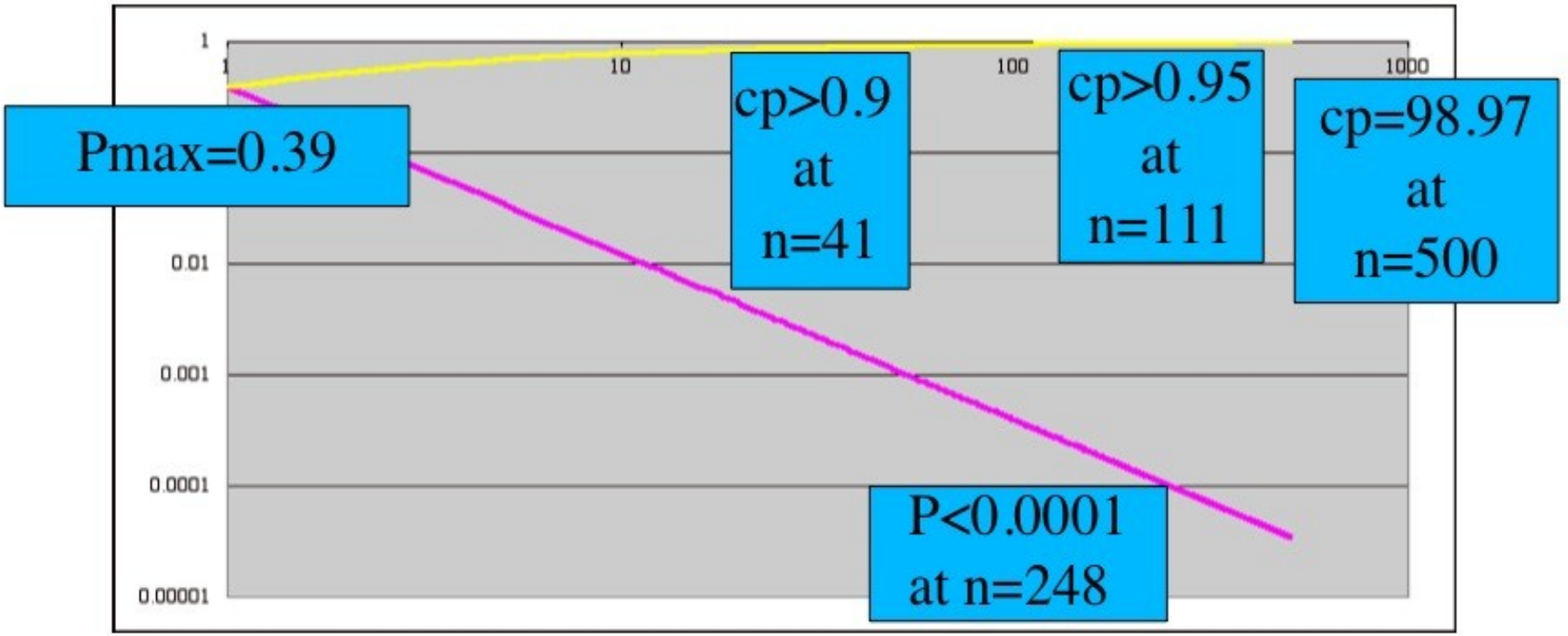
Frequent Word	Number of Occurrences	Percentage of Total
the	7,398,934	5.9
of	3,893,790	3.1
to	3,364,653	2.7
and	3,320,687	2.6
in	2,311,785	1.8
is	1,559,147	1.2
for	1,313,561	1.0
The	1,144,860	0.9
that	1,066,503	0.8
said	1,027,713	0.8

Frequencies from 336,310 documents in the 1GB TREC Volume 3 Corpus
125,720,891 total word occurrences; 508,209 unique words

(from B. Croft, UMass)

Zipf's Law

Probability (purple) and cumulative probability (yellow) of a word appearing in in a document for a 1000 word voculabary with words sorted by their probabilities (a=1.5) (log-log scale)



Pn ~1/(n^a), where Pn is the frequency of occurrence of the nth ranked item and a is close to 1 (best estimates are 1.5-2.0)

Predicting Occurrence Frequencies

- By Zipf, a word appearing n times has rank $r_n = AN/n$
- Several words may occur n times, assume rank r_n applies to the last of these.
- Therefore, r_n words occur n or more times and r_{n+1} words occur $n+1$ or more times.
- So, the number of words appearing **exactly** n times is:

$$I_n = r_n - r_{n+1} = \frac{AN}{n} - \frac{AN}{n+1} = \frac{AN}{n(n+1)}$$

I_n = number of words appearing exactly n times

N = number of words in the collection of words

A = a constant. For example, if N=100 and the most common word appears 10 times then $A = r_n * n/N = 1*10/100 = 0.1$

Note $r_n = AN/n$ and $P_n \sim 1/n^a$ are alternative formulations of Zipf's law

Predicting Word Frequencies (cont)

- Assume highest ranking term occurs once and therefore has rank $D = AN/1$
- Fraction of words with frequency n is:

$$\frac{I_n}{D} = \frac{1}{n(n+1)}$$

- Fraction of words appearing only once is therefore ½.

Occurrence Frequency Data

(from B. Croft, UMass)

Number of Occurrences (n)	Predicted Proportion of Occurrences 1/n(n+1)	Actual Proportion occurring n times I_n/D	Actual Number of Words occurring n times
1	.500	.402	204,357
2	.167	.132	67,082
3	.083	.069	35,083
4	.050	.046	23,271
5	.033	.032	16,332
6	.024	.024	12,421
7	.018	.019	9,766
8	.014	.016	8,200
9	.011	.014	6,907
10	.009	.012	5,893

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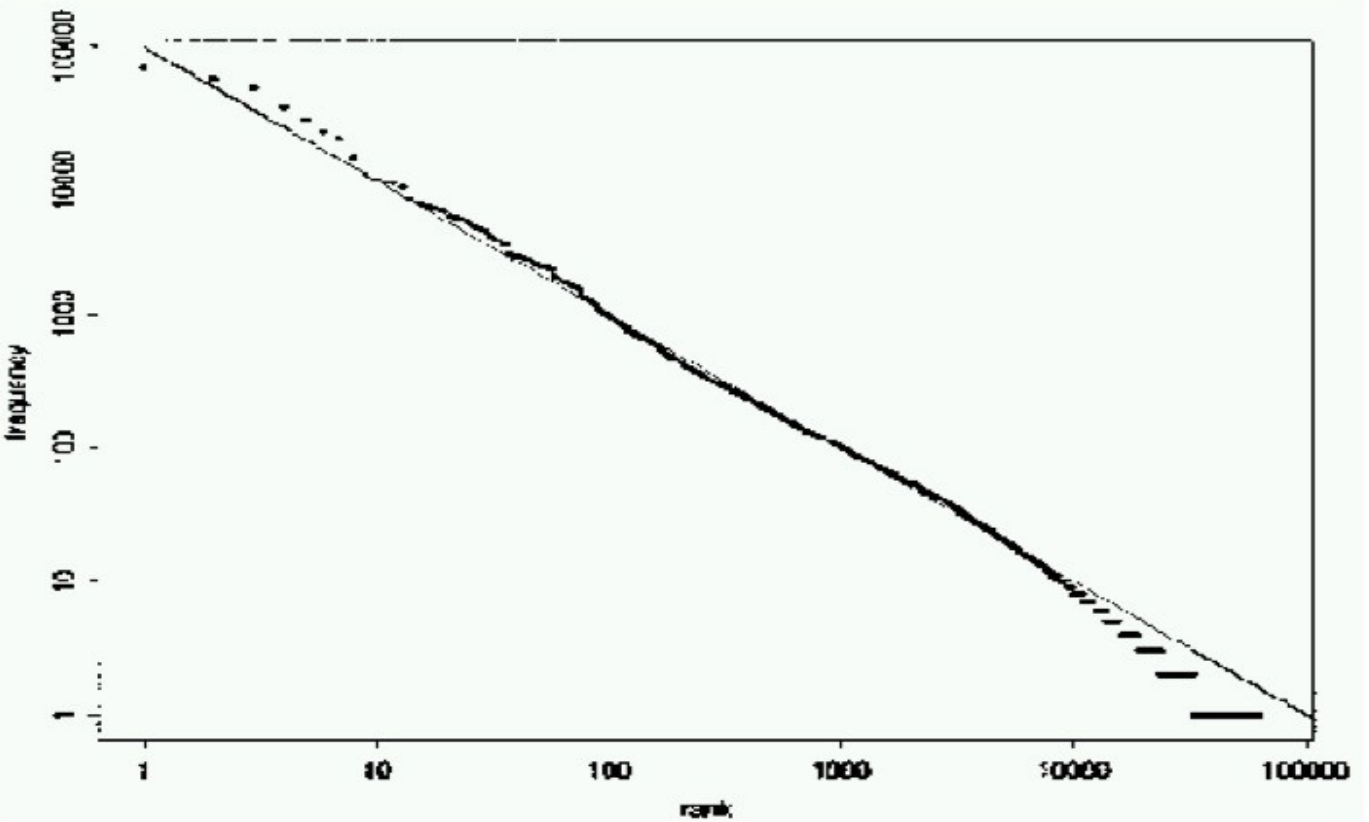
Does Real Data Fit Zipf's Law?

- A law of the form $y = kx^c$ is called a power law.
- Zipf's law is a power law with $c = -1$
- On a log-log plot, power laws give a straight line with slope c.

$$\log(y) = \log(kx^c) = \log k + c\log(x)$$

- Zipf is quite accurate except for very high and low rank.

Fit to Zipf for Brown Corpus



“the” is most common – about 7% , then “of” 3.5% and “and” 2.8%.
Top 135 words account for almost 50% of usage

Explanations for Zipf's Law

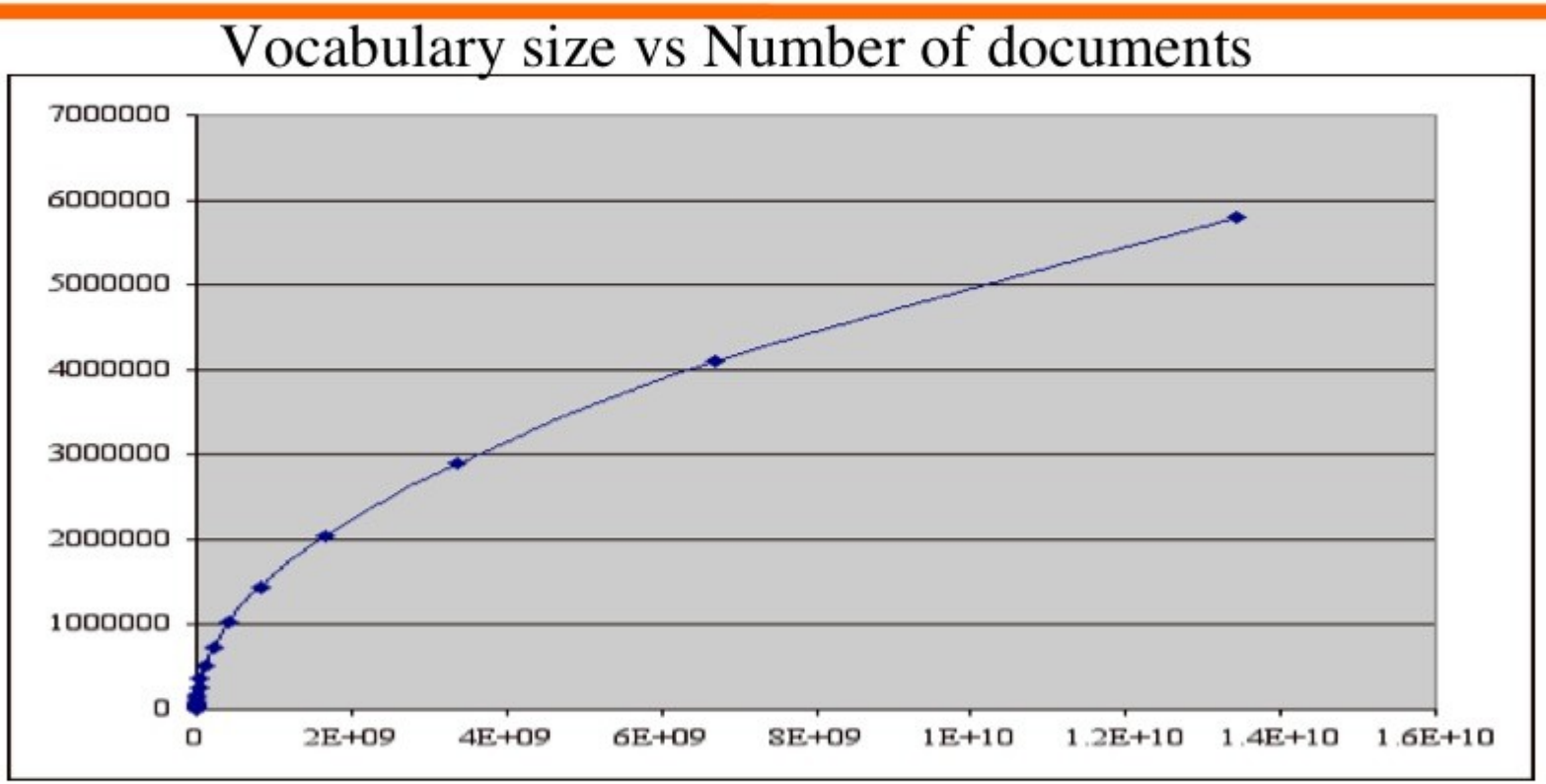
- Zipf's explanation was his "principle of least effort." Balance between speaker's desire for a small vocabulary and hearer's desire for a large one.
- Li (1992) shows that just random typing of letters including a space will generate "words" with a Zipfian distribution.
  - http://linkage.rockefeller.edu/wli/zipf/

Zipf's Law Impact on IR

- Good News: Stopwords will account for a large fraction of text so eliminating them greatly reduces inverted-index storage costs.
- Bad News: For most words, gathering sufficient data for meaningful statistical analysis (e.g. for correlation analysis for query expansion) is difficult since they are extremely rare.

And it Gets Worse.....

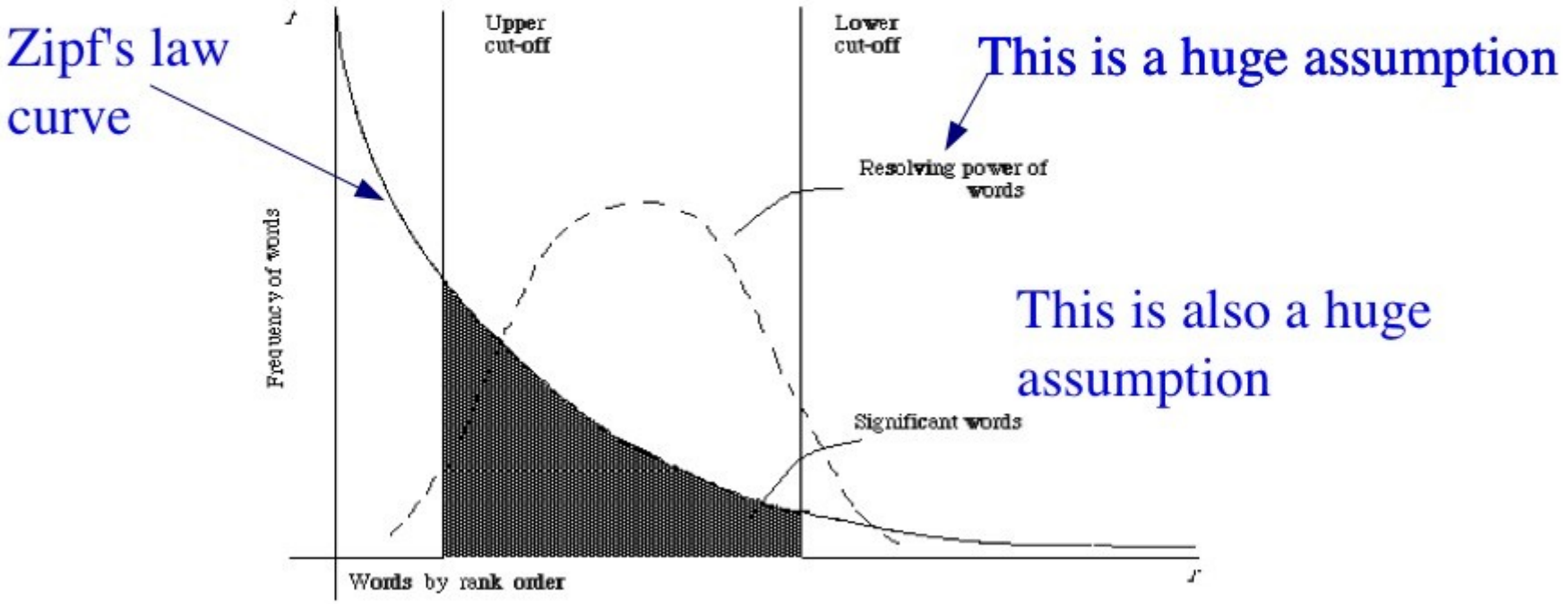
Heap's Law



- V = K n^b
  - b is about 0.5 (according to TREC database), for www b may be bigger
  - K~~10..100
  - So given 1B docs suggest V=1,581,138

But Maybe There is a way out

Escaping Heap and Zipf -- Luhn's Idea



Retrieval Models

- A retrieval model specifies the details of:
  - Document representation
  - Query representation
  - Retrieval function
- Determines a notion of relevance.
- Notion of relevance can be binary or continuous (i.e. *ranked retrieval*).

Classes of Retrieval Models

- Boolean models (set theoretic)
- Vector space models (statistical/algebraic)
  - Generalized VS
  - Latent Semantic Indexing
- Probabilistic models

Other Model Dimensions

- Logical View of Documents
  - Index terms
  - Full text
  - Full text + Structure (e.g. hypertext)
- User Task
  - Retrieval
  - Browsing

Retrieval Tasks

- Ad hoc retrieval: Fixed document corpus, varied queries.
- Filtering: Fixed query, continuous document stream.
  - User Profile: A model of relative static preferences.
  - Binary decision of relevant/not-relevant.
- Routing: Same as filtering but continuously supply ranked lists rather than binary filtering.

Common Preprocessing Steps

- Strip unwanted characters/markup (e.g. HTML tags, punctuation, numbers, etc.).
- Break into tokens (keywords) on whitespace.
- Stem tokens to “root” words
  - computational comput
- Remove common stopwords (e.g. a, the, it, etc.).
- Detect common phrases (possibly using a domain specific dictionary).
- Build inverted index (keyword list of docs containing it).

Boolean Model

- A document is represented as a set of keywords.
- Queries are Boolean expressions of keywords, connected by AND, OR, and NOT, including the use of brackets to indicate scope.
  - [[Rio & Brazil] | [Hilo & Hawaii]] & hotel & !Hilton]
- Output: Document is relevant or not. No partial matches or ranking.

Boolean Retrieval Model

- Popular retrieval model because:
  - Easy to understand for simple queries.
  - Clean formalism.
- Boolean models can be extended to include ranking.
- Reasonably efficient implementations possible for normal queries.

Boolean Models Problems

- Very rigid: AND means all; OR means any.
- Difficult to express complex user requests.
- Difficult to control the number of documents retrieved.
  - *All* matched documents will be returned.
- Difficult to rank output.
  - *All* matched documents logically satisfy the query.
- Difficult to perform relevance feedback.
  - If a document is identified by the user as relevant or irrelevant, how should the query be modified?

Statistical Models

- A document is typically represented by a *bag of words* (unordered words with frequencies).
- Bag = set that allows multiple occurrences of the same element.
- User specifies a set of desired terms with optional weights:
  - Weighted query terms:

    Q = < database 0.5; text 0.8; information 0.2 >
  - Unweighted query terms:

    Q = < database; text; information >
  - No Boolean conditions specified in the query.

Statistical Retrieval

- Retrieval based on *similarity* between query and documents.
- Output documents are ranked according to similarity to query.
- Similarity based on occurrence *frequencies* of keywords in query and document.
- Automatic relevance feedback can be supported:
  - Relevant documents “added” to query.
  - Irrelevant documents “subtracted” from query.

The Vector-Space Model

- Assume *t* distinct terms remain after preprocessing; call them index terms or the vocabulary.
- These "orthogonal" terms form a vector space.

  Dimension = *t* = |vocabulary|
- Each term, *i*, in a document or query, *j*, is given a real-valued weight, $w_{ij.}$
- Both documents and queries are expressed as t-dimensional vectors:

  $d_j = (w_{1j}, w_{2j}, \ldots, w_{tj})$

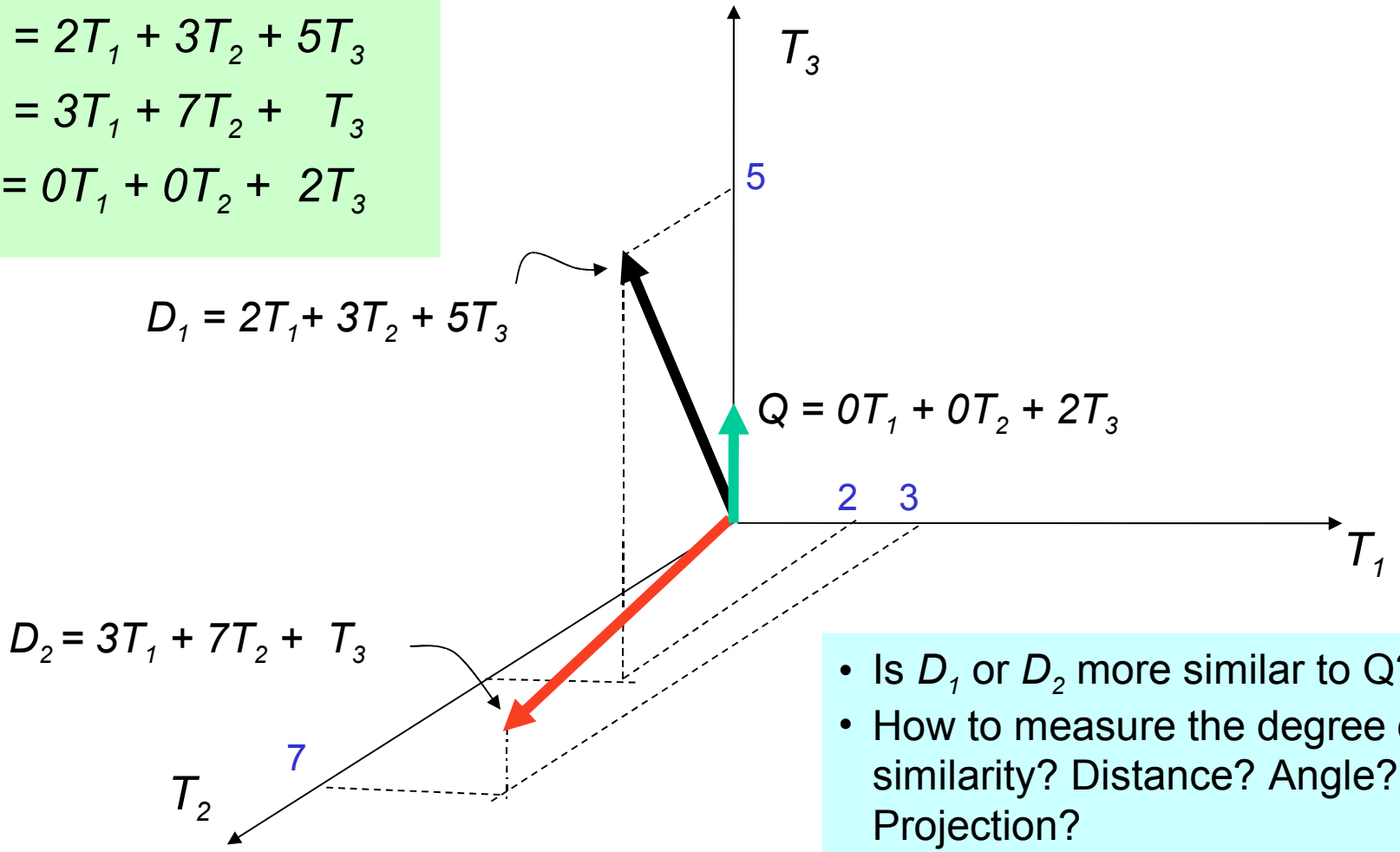
Graphic Representation

Example:

$D_1 = 2T_1 + 3T_2 + 5T_3$

$D_2 = 3T_1 + 7T_2 + T_3$

$Q = 0T_1 + 0T_2 + 2T_3$



- Is D_1 or D_2 more similar to Q?
- How to measure the degree of similarity? Distance? Angle? Projection?

Document Collection

- A collection of *n* documents can be represented in the vector space model by a term-document matrix.
- An entry in the matrix corresponds to the "weight" of a term in the document; zero means the term has no significance in the document or it simply doesn't exist in the document.

$$\begin{bmatrix} & T_1 & T_2 & \ldots & T_t \\ D_1 & w_{11} & w_{21} & \ldots & w_{t1} \\ D_2 & w_{12} & w_{22} & \ldots & w_{t2} \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ D_n & w_{1n} & w_{2n} & \ldots & w_{tn} \end{bmatrix}$$

Term Weights: Term Frequency

- More frequent terms in a document are more important, i.e. more indicative of the topic.

  f_{ij} = frequency of term *i* in document *j*

- May want to normalize *term frequency* (*tf*) across the entire corpus:

  $tf_{ij} = f_{ij} / max\{f_{ij}\}$

Term Weights: Inverse Document Frequency

- Terms that appear in many *different* documents are *less* indicative of overall topic.

  df_i = document frequency of term i

  = number of documents containing term i

  idf_i = inverse document frequency of term i,

  = $\log_2 (N/ df_i)$

  (N: total number of documents)

- An indication of a term's *discrimination* power.
- Log used to dampen the effect relative to *tf*.

TF-IDF Weighting

- A typical combined term importance indicator is *tf-idf weighting*:

$$w_{ij} = tf_{ij}\, idf_i = tf_{ij} \log_2 (N/ df_i)$$

- A term occurring frequently in the document but rarely in the rest of the collection is given high weight.
- Many other ways of determining term weights have been proposed.
- Experimentally, *tf-idf* has been found to work well.

Computing TF-IDF -- An Example

Given a document containing terms with given frequencies:

A(3), B(2), C(1)

Assume collection contains 10,000 documents and

document frequencies of these terms are:

A(50), B(1300), C(250)

Then:

A: tf = 3/3; idf = log(10000/50) = 5.3; tf-idf = 5.3

B: tf = 2/3; idf = log(10000/1300) = 2.0; tf-idf = 1.3

C: tf = 1/3; idf = log(10000/250) = 3.7; tf-idf = 1.2

Query Vector

- Query vector is typically treated as a document and also tf-idf weighted.
- Alternative is for the user to supply weights for the given query terms.

Similarity Measure

- A similarity measure is a function that computes the *degree of similarity* between two vectors.

- Using a similarity measure between the query and each document:
  - It is possible to rank the retrieved documents in the order of presumed relevance.
  - It is possible to enforce a certain threshold so that the size of the retrieved set can be controlled.

Similarity Measure - Inner Product

- Similarity between vectors for the document $\boldsymbol{d}_i$ and query $\boldsymbol{q}$ can be computed as the vector inner product:

$$\text{sim}(\boldsymbol{d}_j, \boldsymbol{q}) = \boldsymbol{d}_j \bullet \boldsymbol{q} = \sum_{i-1}^{t} w_{ij} \cdot w_{iq}$$

  where w_{ij} is the weight of term i in document j and w_{iq} is the weight of term i in the query

- For binary vectors, the inner product is the number of matched query terms in the document (size of intersection).
- For weighted term vectors, it is the sum of the products of the weights of the matched terms.

Properties of Inner Product

- The inner product is unbounded.
- Favors long documents with a large number of unique terms.
- Measures how many terms matched but not how many terms are *not* matched.

Inner Product -- Examples

Binary:

retrieval, database, architecture, computer, text, management, information

- D = 1, 1, 1, 0, 1, 1, 0
- Q = 1, 0 , 1, 0, 0, 1, 1

Size of vector = size of vocabulary = 7
0 means corresponding term not found in document or query

sim(D, Q) = 3

Weighted:

$D_1 = 2T_1 + 3T_2 + 5T_3$ $\quad D_2 = 3T_1 + 7T_2 + 1T_3$

$Q = 0T_1 + 0T_2 + 2T_3$

sim(D_1 , Q) = 2*0 + 3*0 + 5*2 = 10

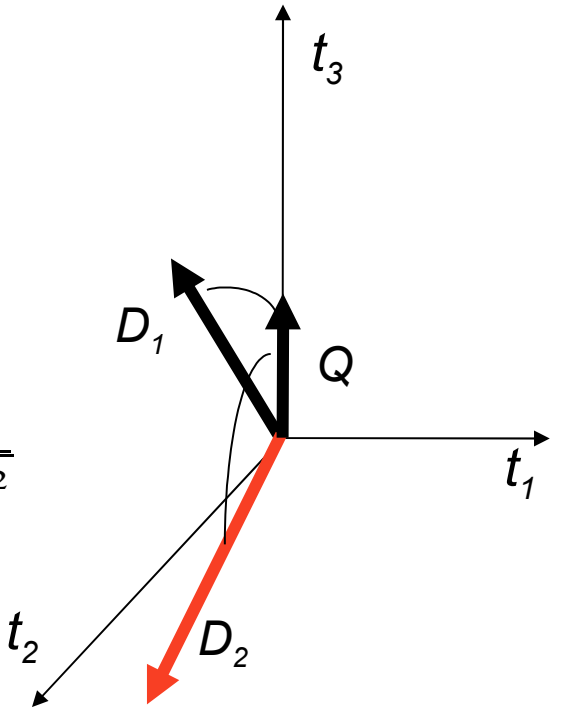
sim(D_2 , Q) = 3*0 + 7*0 + 1*2 = 2

Cosine Similarity Measure

- Cosine similarity measures the cosine of the angle between two vectors.
- Inner product normalized by the vector lengths.

CosSim($\boldsymbol{d}_j$, $\boldsymbol{q}$) =

$$\frac{\vec{d}_j \cdot \vec{q}}{|\vec{d}_j| \cdot |\vec{q}|} = \frac{\sum_{i=1}^{t} (w_{ij} \cdot w_{iq})}{\sqrt{\sum_{i=1}^{t} w_{ij}^2 \cdot \sum_{i=1}^{t} w_{iq}^2}}$$



$D_1 = 2T_1 + 3T_2 + 5T_3$ CosSim(D_1 , Q) = 10 / $\sqrt{(4+9+25)(0+0+4)}$ = 0.81

$D_2 = 3T_1 + 7T_2 + 1T_3$ CosSim(D_2 , Q) = 2 / $\sqrt{(9+49+1)(0+0+4)}$ = 0.13

$Q = 0T_1 + 0T_2 + 2T_3$

D_1 is 6 times better than D_2 using cosine similarity but only 5 times better using inner product.

Naïve Implementation

Convert all documents in collection D to tf-idf weighted vectors, $\boldsymbol{d}_j$, for keyword vocabulary V.

Convert query to a tf-idf-weighted vector $\boldsymbol{q}$.

For each $\boldsymbol{d}_j$ in D do

    Compute score $s_j = \text{cosSim}(\boldsymbol{d}_j, \boldsymbol{q})$

Sort documents by decreasing score.

Present top ranked documents to the user.

Time complexity: $O(|V| \cdot |D|)$ Bad for large V & D !

$|V| = 10{,}000$; $|D| = 100{,}000$; $|V| \cdot |D| = 1{,}000{,}000{,}000$

Comments on Vector Space Models

- Simple, mathematically based approach.
- Considers both local (*tf*) and global (*idf*) word occurrence frequencies.
- Provides partial matching and ranked results.
- Tends to work quite well in practice despite obvious weaknesses.
- Allows efficient implementation for large document collections.

Problems with Vector Space Model

- Missing semantic information (e.g. word sense).
- Missing syntactic information (e.g. phrase structure, word order, proximity information).
- Assumption of term independence (e.g. ignores synonomy).
- Lacks the control of a Boolean model (e.g., *requiring* a term to appear in a document).
  - Given a two-term query “A B”, may prefer a document containing A frequently but not B, over a document that contains both A and B, but both less frequently.